OXFORD

Open-ended Maths Activities

Using 'good' questions to enhance learning in Mathematics

Second Edition

Peter Sullivan and Pat Lilburn

NUMBER

The six topics included in this strand are:

1 Money
2 Fractions
3 Decimals
4 Place Value
5 Counting, Pattern and Order
6 Operations

There are links in these number topics with the other areas of the mathematics curriculum and with each other. It is neither possible nor useful to try to treat them separately. The questions in each topic do, however, have their main teaching point within that topic.

While answering these questions children will develop a feeling for the way numbers work. They will develop a strong sense not only for whole numbers but also for where fractions and decimals fit into the number system. They will understand the importance of estimation and mental calculation skills and use calculators to enable them to understand key ideas without having to do complicated calculations before they are ready to do so.

Do not forget to adapt questions where necessary by making numbers or amounts smaller or larger.

MONEY (Junior)

Experiences at this level will help children to:

- recognise different notes and coins
- describe, sort and classify coins and notes
- exchange money for goods in play situations and give appropriate change
- order money amounts
- use coins to represent written money amounts and use numbers to record the value of a group of coins
- use estimation and a calculator for money calculations.

Materials needed:

- play money, coins and notes
- 'goods' marked with varying prices. (Ensure that there are combinations of items that add to $1.)

'Good' questions and Teacher notes

1 **How many different ways can you make 20c?**

2 **In my pocket I have 75 c. What coins might I have?**

Q1 & Q2 Children should realise that there are many different ways to make a money amount. See if they use multiples of one coin, e.g. 4 × 5c, as well as different coins, e.g. 10c + 5c + 5c. Notice if they develop a systematic way to find coin combinations, e.g. for 75c they might start with 15 × 5c, then 7 × 10c + 5c, then 3 × 20c + 10c + 5c, etc.
Are children confident when counting in 5s, 10s, 20s?

3 **Choose an amount of money, for example, 75c, $1.20 .... Ask children to choose an item from some items on display that they can afford to buy with the amount of money. Ask them to choose an item that they cannot afford to buy with the amount of money.**

You will need a variety of items each with a price tag showing.

4 **I bought something at a supermarket and got 5c change. How much did it cost and how much money did I give to pay for it?**

Children should develop a system. Some may do it randomly to begin with and then realise the pattern, e.g.:

- costs 5c and gives 10c
- costs 15c and gives 20c
- costs 95c and gives $1
- costs $1.95 and gives $2.

Can children see the folly of giving 15c for an item costing 10c to receive 5c change?

5 **I spent exactly $1 at our class shop. What might I have bought?**

Check how easily children can add amounts to $1. They should be able to mentally estimate by rounding before writing down amounts.

Note if they calculate multiples of 5, 10 or 20 to make $1, e.g. do they know five items at 10c each is 50c or do they add each one separately?

6 **I am a coin with an animal on me that does not swim. What might I be?**

The main focus here is to look more closely at the attributes of coins.

7 **I have two coins in one hand and one in the other hand. Each hand is of equal value. What could the coins be?**

Note if children develop a system when recording. How easily do they calculate amounts?

8 **The answer to a calculation is 35c. What is the question? Refer to this list to help you.**

CANTEEN PRICE LIST

Vegemite sandwich	$1.10	Salad sandwich	$1.55
Ham & salad roll	$1.40	Bag of chips	65c
Fruit salad	$1.15	Piece of fruit	20c
Cookie	15c		

Can children write more than one question? Do they limit their questions to addition or c they make others?

9 **I had one of each of the coins in our currency on my table. I sorted them into two groups. Wh might the groups have been?**

It is interesting to note what categories children use. Ask them to tell you their categori don't assume you know their reasoning.

10 **If I have three Australian coins in my pocket, how much money might I have?**

Ask students to be systematic, and to find out how many different amounts they might ha

11 **The price tag on a toy car is $2.75. What coins would I use to pay for this?**
Note if children develop a system when recording. How easily do they calculate amounts?

12 **I have exactly $100 in notes in my pocket. What notes might I have?**
Check how easily children can count in 5s, 10s, 20s, 50s.
Are they aware of available notes? Are they systematic in recording?

13 **Someone was asked to remember the cost of five items. They knew the most expensive was $2 and the least expensive was 50c. What might the other three be?**
The focus here is on ordering of money amounts. Note if the children record different amounts correctly.

MONEY (Middle)

Experiences at this level will help children to:

- round to the nearest dollar to estimate or check total cost
- record money amounts
- tender appropriate amounts when the exact amount is not available
- order money amounts
- use an appropriate method (mental, written, calculator) to solve problems involving money and processes.

Materials needed:

- play money, notes and coins
- supermarket catalogues
- calculators.

'Good' questions and Teacher notes

1 **I bought an item at a shop and got 35c change. What did I buy and how much did it cost?**
Children need to see the folly of including such things as buying an item costing 5c and giving 40c to get 35c change.
Note if children look for a pattern when recording answers.

2 **I gave change of $1 using 20c, 10c and 5c coins. What might the change have looked like?**
Note if children record systematically and accurately.
How easily do they deal with addition of 20, 10 and 5 and multiplication of each of these to make them up to $1?

3 **Show some different ways to give change from $2 for an item costing $1.35.**
Allow children to use play coins. Do they systematically record the different ways?

4 **How could I spend exactly $20 at the supermarket? (Use a catalogue and a calculator to help.)**
Check if children use estimation skills to help them. They might round off some amounts to assist their estimation. Note how they use the calculator.

5 **In my pocket I have $36. What notes and coins might I have?**
This allows you to see how familiar children are with the various notes and coins and if they use a system when recording.

6 **If I have three Australian notes in my wallet, how much money might I have?**
Ask students to be systematic, and to find out how many different amounts they might have.

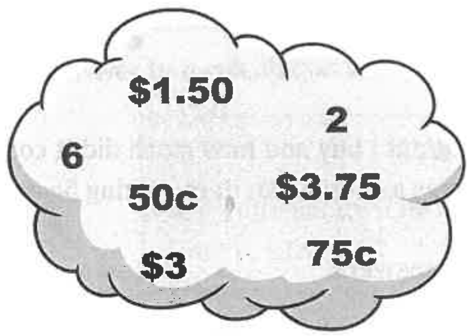
7 **I spent $60 on six tickets to the theatre. How many adults and children are there and how much are the tickets?**
Are the answers realistic? Can children multiply amounts, e.g. 4 × $10 or 4 × $5? Note how they do this, i.e. mentally, with a calculator, etc.

8 **I looked in the paper and saw five cars. The most expensive was $12 150, the cheapest was $9 700, and the one in the middle was $10 000. What prices might the two missing cars be?**
The focus here is on ordering of money amounts. How easily can children work with numbers above and below 10 000?

9 **When I was in a music shop I saw that a CD cost about $22 and a tape about $15. What might have been the price tag on the CD and the tape?**
This question focuses on rounding off. Are children aware that they can round up and down?

10 **Prices in supermarkets are to the exact cent even though we do not have 1c and 2c coins. I bought two items and was asked to pay $4.65. What might have been the price of each item?**
See if children realise that the two amounts do not have to total $4.65 exactly, i.e. one could be $1.22 and the other $3.41, which totals $4.63 and rounded off makes $4.65.

11 **A number sentence uses three of the amounts or numbers in this cloud.**



What might the number sentence be?
The main focus here is to see if children use a variety of processes, e.g. 6 × 50c = $3, $1.50 ÷ 2 = 75c, $3.75 – 75c = $3.

12 **My friends and I shared an amount of money equally between us. We each got $1.20. How much money was there and how many friends might I have?**
It is interesting to see how children do this — mentally, by writing down or with coins. When they check their answer do they include themselves or only the friends?

13 **I bought something and paid for it with three coins. What might it have been and how much did it cost?**
Look for a range of responses that are realistic.

14 **I went to get $100 out of the bank. What are the different ways I can ask for this amount of notes?**
How easily can children multiply and divide by 2, 5, 10 and 20?

MONEY (Upper)

Experiences at this level will help children to:

- use mental calculation and estimation
- use +, –, × and ÷ for written computation of money
- select an appropriate operation to solve problems involving money.

Materials needed:

- play money, notes and coins
- new and used car section of a newspaper
- current exchange rates
- calculators.

'Good' questions and Teacher notes

1 **Scientific calculators cost $40 and basic calculators cost $12. How much might it cost for a class set of some basic and some scientific calculators?**

Note how children decide how many of each calculator to purchase. Do they record their answers systematically? Do they choose appropriate operations to work out the price? How easily do they handle these operations?

2 **I have $14 000 and want to buy two cars. What could I buy?**

Note if children can justify their answers and if they can provide a range of answers.

3 **If one of the notes currently in use was to be changed to a coin, which one would you choose? Why?**

Children should be able to justify their choice in a reasonable manner. You could extend this by looking at notes and coins in use in other countries.

4 **You are spending five nights away. You have won $500 for accommodation. Where could you stay?**

Top class hotel	$300 per night
4 star hotel	$225 " "
3 star hotel	$100 " "
2 star hotel	$60 " "
Backpackers	$25 " "

Note what methods children use to work this out, i.e. do they readily multiply amounts when needed or do they always add amounts? They can stay at different places.

5 **Imagine that you are in some other country. A hamburger costs 10 units of that country's money. What country might you be in?**

Students can use the web or newspaper to find out information about exchange rates.

6 **Design a rounding policy for a supermarket.**
The way children approach this will tell you if they understand rounding. It is interesting to note whose side they are on — owner or customer?

7 **I have more than $10 in coins, but I cannot change a $10 note into coins exactly. How much money might I have?**
It is a good idea to let children use play coins. This is a hard question. You could hint that when we used 1c and 2c coins, if you had 3 × 2c coins you could not change 5c.

FRACTIONS (Junior)

Experiences at this level will help children to:

- use informal fraction language for objects and collections
- compare fractional parts of objects and collections.

Materials needed:

- fraction materials such as rods, counters, shapes, etc.
- paper squares
- lengths of string, tape, sticks, etc.

'Good' questions and Teacher notes

1 **You see a sign in a shop window $\frac{1}{2}$ PRICE SALE. What does this mean to you?**
Listen carefully to children's responses as these will indicate the depth of their understanding. You could extend them by giving them some examples of prices.

2 **Half of the people in a family are males. What might a drawing of the family look like?**
Do children understand that there must be the same number of people in each group? Make sure they see a range of drawings done by class members.

3 **Draw a shape. If the shape is cut into two halves what might it look like?**
Do children show equal parts?

4 **How many different ways can you cut a square into quarters?**
Check if children know that each piece has to be the same size. You could do this activity with other shapes and other unit fractions.

5 **Draw some shapes that are divided into two parts, but the parts are not halves. Draw some groups that are divided into two parts, but the parts are not halves.**
Ask children to justify why the parts they draw are not halves.

6 **I was listening to the radio and I heard the announcer say 'half'. What might she have been referring to?**
The answers will indicate depth of understanding. Two possible responses are 'Half-past 3' and 'Half-time'.

7 **We want to paint the top half of the room. How could we find out where the half-way mark is?**
Allow children to do this how they want but have some string, tape, etc. available for their use. The main focus here is to look at how practical the strategies are. Accuracy is not important.

8 **What do you know and what can you find out about $\frac{1}{4}$? Record it on paper or show it with materials.**
Note if children understand $\frac{1}{4}$ as part of a whole and as part of a collection. Can they use a range of materials to represent it?

FRACTIONS (Middle)

Experiences at this level will help children to:

- represent simple fractional parts of objects and collections
- order and compare fractions with related denominators
- express common fractions
- record simple equivalence, e.g. $\frac{1}{2} = \frac{2}{4} = \frac{3}{6}$
- add and subtract tenths and fractions with like denominators.

Materials needed:

- fraction materials, e.g. counters, shapes, rods, etc.
- drawing paper for designs, kindergarten squares, paper circles
- circles cut into quarters to represent 'pizzas'.

'Good' questions and Teacher notes

1 **How many different designs can you make that are $\frac{3}{4}$ red and $\frac{1}{4}$ yellow?**
Note if the designs are simple or complex. Are they creative? Ask children to explain how they know $\frac{3}{4}$ is red and $\frac{1}{4}$ is yellow.

2 **$\frac{3}{8}$ of a circle is coloured red. I cannot remember what colours the rest of the circle was. What might the circle look like?**
Provide circles but allow children to divide them into eighths. This will show you if they understand the concept of equal parts. They may colour the other parts any colour or colours they choose.

3 **One-third of a class order lunches from the canteen each day. How many students might be in the class and how many of them order lunches each day?**
Let children use counters to represent the students if they wish. Do the children base their answers on multiples of 3?

4 **My friend and I ate all of a pizza which was cut into 8 equal pieces. What fraction of the pizza might each of us have eaten?**
Note children who work this out systematically.

5 **My aunt said that when she was half her age she could touch her toes. How old might she be now and how old was she when she could touch her toes?**

This requires children to use $\frac{1}{2}$ as an operator. Check that suggested answers are realistic.

6 **I picked up a handful of Smarties. One-third of them were red. What might a drawing of the Smarties look like?**

This requires children to show a part of a collection. Are the parts shown equal? Do children provide a range of answers and does anyone develop a system to do so? Do children understand why amounts that are not multiples of 3 do not work?

7 $\frac{\square}{5} + \frac{\square}{5} = \frac{\square}{5}$ **What might the missing numbers be?**

Do children realise there is more than one possible answer?

8 **My big brother has received six test results so far. On five of them he scored half marks. What might his results have been?**

The focus here is on halving and doubling. How creative are children with their answers? Do some children insist that you cannot halve an odd number?

9 **I had some pizzas that I cut into quarters. How many pizzas might I have had, and how many quarters might I have after cutting them?**

Can children identify a pattern? Do they see the relationship between improper fractions and mixed numbers?

10 $\frac{1}{\square} = \frac{2}{\square} = \frac{3}{\square}$ **What might the missing numbers be?**

Do children develop a pattern when answering this? Can they continue the pattern? If 10 was the numerator for their pattern what would be the denominator?

11 **How many different ways can you show $\frac{2}{3}$?**

Note if children understand $\frac{2}{3}$ as part of a whole or part of a length and as part of a collection. Do they use a range of materials to represent it?

12 **I folded a kindergarten square to show a fraction. How did I fold it and what might the fraction have been?**

Look for equal parts and a range of answers.

13 **Someone was counting by fractions and the last thing they said was '10'. What might the four numbers before this have been?**

How easily can children choose fractions before 10 in order, e.g. 10, $9\frac{3}{4}$, $9\frac{2}{4}$, $9\frac{1}{4}$, 9 or 10, $9\frac{9}{10}$, $9\frac{8}{10}$, $9\frac{7}{10}$, $9\frac{6}{10}$? Do they provide a range of answers?

14 **A friend of mine put these fractions into two groups, but they got mixed up. What might the two groups be?**

$$\frac{2}{5},\ \frac{3}{4},\ \frac{6}{10},\ \frac{1}{3} \text{ and } \frac{1}{10}$$

Ask children to give reasons for their groups as these could highlight some misconceptions. One possible grouping is to put $\frac{1}{3}$ and $\frac{1}{10}$ in one group because they are unit fractions; another grouping is to put $\frac{3}{4}$ and $\frac{6}{10}$ in one group because they are bigger than $\frac{1}{2}$.

FRACTIONS (Upper)

Experiences at this level will help children to:

- use equivalence to compare and order fractions
- locate and count fractions on a number line
- rename fractions in different forms, e.g. as percentages or decimals
- mentally add and subtract common equivalent fractions
- add and subtract fractions with related denominators
- understand the relationship between division and fractions.

Materials needed:

- fraction materials, e.g. counters, shapes, rods, kits, etc.

Number

'Good' questions and Teacher notes

1 **Two fractions add to give $\frac{1}{2}$. What might those two fractions be?**
Do children only use known fraction combinations such as $\frac{1}{4} + \frac{1}{4}$ or do they use subtraction to find other possibilities, e.g. $\frac{1}{2} - \frac{1}{3} = \frac{1}{6}$, so $\frac{1}{3} + \frac{1}{6} = \frac{1}{2}$? Do they use equivalence, e.g. $\frac{1}{2} = \frac{6}{12}$, so $\frac{1}{12} + \frac{5}{12} = \frac{1}{2}$?

2 **Some numbers add to give 10. I know that at least one of them has a fraction part in it, but none uses decimals. What might the numbers be?**
As above, note the methods children use to find answers. These will tell you a lot about their understanding of fractions.

3 **$\frac{1}{2} = \frac{\square}{\square} + \frac{\square}{\square} + \frac{\square}{\square}$ What might the missing numbers be?**
Again look at the methods used. Do children guess and then work it out to check? If so, how do they then adjust the fractions? Do they know which fractions are smaller than $\frac{1}{2}$?

4 **The answer is $\frac{3}{7}$. What might the question be?**
Encourage children to use other processes than just addition.

5 **Using proper fractions record some fraction additions where the answer will be more than 1. Also record some fraction additions where the answer will be less than 1.**
Proper fractions are of the type $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{5}$ … where the numerator is smaller than the denominator.
The purpose of this is for children to develop strategies for knowing if the answer will be more of less than 1 without having to calculate the answer. For example, $\frac{8}{11} + \frac{3}{5}$ has to have an answer larger than 1 because both fractions are larger than $\frac{1}{2}$.

6 **What fractions have a difference of $\frac{3}{4}$?**
Do children use equivalence ('I know $\frac{3}{4} = \frac{6}{8}$, so $\frac{7}{8} - \frac{1}{8} = \frac{3}{4}$') or some other method to do this?

7 **$\frac{1}{\square} \times 3\square = 1\square$ What might the missing numbers be?**
The missing numbers do not have to be the same. Can children describe all of the answers? Can they prove they have all of the answers? (There are nine possibilities.)

8 **Teacher: Tell me a fraction between $\frac{1}{2}$ and $\frac{3}{4}$.**
Student: $\frac{2}{3}$, because 2 is between 1 and 3, and 3 is between 2 and 4.
Is this reasoning always correct? If not, give examples where it does not work.
This question highlights a misconception that some children may have.

9 **A rectangle has a perimeter of two units. What might the area be?**
Yes, this is a fraction question! The perimeter must be a combination of fractions, e.g. $\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2}$, $\frac{1}{4} + \frac{3}{4} + \frac{1}{4} + \frac{3}{4}$, etc. The area will vary depending on the length of the sides. You may want to remind students that a square is a rectangle.
You could ask children why the area is a smaller fraction than the sides. When you multiply two fractions together is the answer always smaller?

10 **Write some different stories about $3 \div \frac{1}{2}$.**
The purpose of this is for children to understand the difference between $3 \div \frac{1}{2}$ and $\frac{1}{2}$ of 3. Their stories will indicate this. Stories like 'I had $3 and I gave half to my friend' are not appropriate.

11 **Teacher: 'Which is bigger, $\frac{201}{301}$ or $\frac{2}{3}$?'**
Student: '$\frac{201}{301}$ is bigger because 1 has been added to the top and the bottom.'
Is this reasoning correct? Are there any examples where adding 1 to the top and the bottom makes the fraction bigger?
This question highlights a misconception that some children may have.

12 **$\frac{\square}{\square} < \frac{3}{4}$ What might the missing fraction be?**
Provide concrete materials for this question. Do not assume that because some children write, for example, $\frac{1}{3}$ that their understanding is correct. Check why they write this. Some children may think any number smaller than the 3 or the 4 will make a smaller fraction and will not consider fractions such as $\frac{1}{5}$, $\frac{5}{10}$, etc. to be smaller. This question checks the same misconception as Q11.

13 **What might the missing numbers be?**
$\frac{\square}{\square} + \frac{\square}{\square} = \frac{2}{5}$
Do children use fractions other than those with a denominator of 5?

14 **What might the missing numbers be?**
$\frac{\square + 1}{\square + 1} < \frac{2}{5}$
How do children find fractions smaller than $\frac{2}{5}$? Who words backwards by finding a fraction less than $\frac{2}{5}$ and then subtracting 1 from the numerator and denominator? For example: $\frac{3}{8} < \frac{2}{5}$ so the numerator would be 2 + 1 and the denominator would be 7 + 1.

15 **$1\frac{\square}{\square} = \frac{\square}{\square}$ What might the missing numbers be?**
Can the children draw a link between mixed numbers and other fractions? Can they describe how they found an answer?

16 **Write down ten fractions between $\frac{1}{3}$ and $\frac{2}{3}$.**
This question can also be posed as $\frac{1}{3} < \frac{\square}{\square} < \frac{2}{3}$ and also as $\frac{1}{3} < \frac{\square}{\square} < \frac{\square}{\square} < \frac{\square}{\square} < \frac{2}{3}$

DECIMALS (Middle)

Experiences at this level will help children to:

- read decimals on a calculator screen
- express decimal fractions using appropriate notation
- round off to nearest whole number
- record and order numbers with two decimal places
- add and subtract numbers involving tenths
- understand how decimals fit in the number system.

Materials needed:

- calculators
- materials to model decimals, e.g. base 10 materials, abacus, bundled sticks, etc.
- dice.

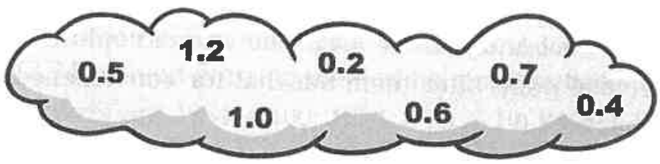
'Good' questions and Teacher notes

1 **A decimal number has been rounded off to 6. What might the number be?**
This will establish if children understand the concept of rounding. Do they give one number or do they know it can include 5.5 but must be smaller than 6.5?

2 **□.□ + □.□ = 5 What might the missing numbers be?**
Do children realise that 5 is the same as 5.0? Check that they can give a range of answers.

3 **Players in turn roll a dice (ideally, it can be a 10-sided dice, but a 6-sided dice also works) and write the number in one of the squares. □.□ + □.□ =10**
The aim is to make the answer to the addition as close as possible to 10. Once the number has been written, it cannot be erased or rewritten.
This is as much a place value question as a decimal question. The way children approach this will indicate if they understand the value of the places.

4 **How many different equations can you make using the numbers in this cloud?**



Look for a variety of operations.

5 **I am thinking of some decimal numbers between 1 and 2. What might they be? Give at least 15 answers.**

6 **My big sister says that the 100 m record at her school is between 12 and 13 seconds. What might the record be?**
Q5 & Q6 Note which children give answers using only hundredths or only tenths and which children give a combination of these. Are there any children who include thousandths? Does anyone give the complete range of tenths and hundredths?

7 **Using only these buttons on your calculator [5] [.] [4] [+] [=] what numbers can you make t calculator show?**

Children should use a range of processes. Note how comfortable they are with functioning of the calculator.

8 **Represent 1.4 in at least five different ways. Show the models.**

Provide plenty of concrete materials for this question. Some you might find useful are abacus, base 10 blocks, bundled sticks, metre rulers and 10 cm rulers, play moı centicubes.

9 **The difference between two numbers is 0.3. What might the two numbers be?**

Can children give many answers? It is interesting to note which children give examples t bridge whole numbers, i.e. 3.2 – 2.9 = 0.3 and which children do not include examples of type. Do they notice a pattern?

10 **What might the missing numbers be?**

□.□ + 0.□ = 4.1

Can students give all possible solutions? Do they record their answers systematically?

11 **I added three decimal numbers together to make exactly 4. What might the three numbers**

Look for a variety of answers. Do children add two of them and then subtract from 4 to the third?

12 **One of your friends, Fred, asked you to help him with his addition calculations. He has done some questions like this:**

1.1 + 0.2 = 1.3
2.3 + 4 = 2.7
6 + 3.7 = 9.7
6.9 + 2 = 7.1

What are some other questions that Fred might get wrong? What advice would you give Fr to help him?

The questions that Fred might get wrong are those where a whole number is added decimal number and the decimal number is written first. Some other examples migh 5.4 + 3 = 5.7, 2.8 + 6 = 3.4 and 7.5 + 4 = 7.9 The advice could include the fact that w numbers have a decimal point after them but that for convenience we do not write it. example, 4 is the same as 4.0

13 **If I use a MAB 'flat' to represent units, a 'long' to represent tenths, and a 'mini' to represeı hundredths what numbers can I represent using exactly ten pieces?**

Children need to work with MAB base 10 to do this question. You could extend it by as them to show the biggest or smallest possible number using ten pieces.

14 **In this number sentence, what might the missing digits be? 2 < □.□**

Can children give the entire range (2.1 to 9.9, including numbers like 5.0)?

15 **□.□ × □ = □.□ What might the missing numbers be? Use a calculator to help you.**
This question is easier if children look for a pattern. Ask them to discuss rules which might help them understand multiplication using decimals.

16 **SNOLITS are words that have the 'same number of letters in their spelling'. For example the word 'twenty-two' has 9 letters and the word 'thirty-two' has 9 letters so 22 and 32 are SNOLITS. The word 'twelve' has 6 letters and the word 'fifteen' has 7 letters so that pair of numbers are not SNOLITS.**
A number, and the number 1.1 more than it, have the same number of letters in their spelling. What might the numbers be?
Before children start this explain that when writing 1.1 in words, that is 'one point one', we count point as 5 letters so 1.1 has 11 letters in its spelling.

DECIMALS (Upper)

Experiences at this level will help children to:

- read, record and order decimals to three places, including decimal fractions with an unequal number of places
- round off to the nearest whole number and use this skill to estimate
- rename common fractions as decimals and vice versa
- count by decimal fractions
- select and use appropriate operations to deal with decimals and percentages
- compare and order percentages, decimals and fractions
- calculate percentages of numbers and quantities.

Materials needed:

- calculators
- metre rulers
- strips of card or paper.

'Good' questions and Teacher notes

1 **How many different ways can you make your calculator show a number with a particular decimal such as 12.34 without pressing the decimal point button?**
Some possible answers are 1234 ÷ 100; 1234 ÷ 10 ÷ 10; 2468 ÷ 200; 617 ÷ 50; 1234 × 1% (if available).

2 **In a timber yard I found 15 pieces of timber, all different lengths but all between 2.1 and 2.2 m long. How long might each piece of timber have been?**
Children may respond with 2.11, 2.12, 2.13 … 2.19, and then not know any others. To help, ask them if there are any numbers between 2.11 and 2.12. Maybe they could represent the lengths of timber with card or paper strips.
Some children might insist that 2.10 and 2.20 are different from 2.1 and 2.2.

3 **Write down ten numbers between 3.01 and 3.1**
To get ten numbers children have to include at least two decimal numbers with thousandths.

4 **Two numbers each with four digits are added and the result is rounded off to 2.7. What might those numbers be?**

The two numbers must total in the range 2.650 to 2.749 to be rounded off to 2.7. Do any children work out this 'rule' before trying to find the two numbers?

5 **In this calculation some numbers are missing. What might they be?**

```
    3 . ❒
+  ❒ . 7 ❒
-----------
    6 . ❒ 3
```

Do children give more than one answer and do they record their answers systematically? Do they realise that ten different digits can go on the top line?

6 **When writing a sequence of numbers I wrote down 2.57 to start and 3.61 to finish. What might the numbers in-between be?**

The children could count by 0.02, 0.04, 0.08 or 0.26. Note which of them use a calculator to assist them to work this out. Do any children find the difference between 3.61 and 2.57 (1.04) and then find out which numbers divide evenly into it?

7 **3.❒1 + ❒.47 + 0.❒ = 8.68 What might the missing numbers be?**

Note how children do this. Who does it by trial and error? Who develops a system such as deciding on one of the numbers and then subtracting it from 8.68 to see what is left to work with?

8 **Two numbers multiply together to give 14.4. What might these numbers be?**

It is useful if children have calculators to help with this one. Do they divide 14.4 by various numbers to find a solution, e.g. $14.4 \div 2.5 = 5.76$, so $5.76 \times 2.5 = 14.4$?

9 **I multiplied 15 by a decimal and got an answer less than 15. What might be the decimal that I multiplied 15 by? Give at least 10 possibilities.**

Can children generalise that the decimal fraction has to be less than 1?

10 **The area of a rectangle is 0.9 square units. What might the perimeter be?**

This is similar to Q8. By using a calculator children can discover many possible answers, e.g. $0.9 \div 0.25 = 3.6$, so two sides of the rectangle could be 0.25 and 3.6, which means the perimeter would be 7.7.

You could ask the children to find the largest or smallest perimeter to encourage them to find a pattern.

11 **What numbers could be rounded off to 5.8?**

This gives children a different way to think about rounding and emphasises the significance of place value. The numbers can include or be bigger than 5.75 but must be less than 5.85. Most children can give one number that can be rounded. If they can give the entire range it shows they have a complete understanding of the concept of rounding.

12 In a race the times are measured to hundredths of a second. The winner's time is 12.52 seconds. What might the times of the other eight runners be?

The main point is to check if children are able to order decimals. Note how realistic the times are. The times of the other runners must be greater than 12.52, e.g. 12.53, 13.27, etc.

13 ❒.❒ × ❒.❒ = ❒.❒ What might the missing numbers be? Use a calculator to help you.

Do children try to do this by trial and error or do they know that they must multiply numbers with 0.2 and 0.5 to get a result with only tenths not hundredths? Some possible answers are $1.2 \times 1.5 = 1.8$, $1.2 \times 3.5 = 4.2$ … $1.2 \times 6.5 = 7.8$; $2.2 \times 1.5 = 3.3$, $2.2 \times 2.5 = 5.5$ … $2.2 \times 4.5 = 9.9$, and so on. It is important to see the pattern if you want to find all possible answers.

14 I divided 6.12 by 3 and wrote down the answer, 2.4. What did I do wrong and what other similar questions might I get wrong?

It is a common mistake for children to leave out the zero in such examples. If they are able to give other examples then they are aware of the error.

15 I divided a whole number by a decimal fraction and got an answer of 24. What might the number sentence look like?

Some possibilities are: $12 \div 0.5$, $6 \div 0.25$, $18 \div 0.75$, $3 \div 0.125$, $9 \div 0.375$

16 In a school the difference between the tallest and the shortest teacher is 0.35 m. What might the heights of the tallest and shortest teacher be?

Do children know 0.35 m is 35 cm? Can they give some realistic responses?

17 I converted a common fraction to a decimal using division. It has a pattern like 0.abababab… What might the fraction be?

Children should use a calculator to do this. The numerator can be numbers other than 1. Can they find other patterns during their investigation?

18 How can you work out a way to get the answer for 3.5 ÷ 4 without pressing the 5 button on your calculator?

Help each child explain their strategy and what it means. The answer is 0.875. One way of doing it is to do $3.6 \div 4 = 0.9$ and then subtract the result of $0.1 \div 4$ (0:25) from 0.9 to make 0.875.

19 1.❒ × 1.❒ = 2.❒ What might the missing numbers be?

The key point is that a zero has been dropped off at the end.

20 I multiplied two decimal numbers on a calculator and got a whole number answer. What might the two decimal numbers have been?

This is similar to the previous one. The answer will have at least two zeros at the end, which have been 'dropped off' by the calculator.

21 Sue was given 20% discount on the price of a tennis racquet. How much did the racquet cost, and how much did Sue pay?

Children must use realistic prices. You might like to investigate prices of racquets.

PLACE VALUE (Junior)

Experiences at this level will help children to:

- use place value to recognise, order and record three-digit numbers
- develop early rounding ideas to 10
- recognise zero as a number
- make models of three-digit numbers
- rename numbers up to 1000
- group items in tens to count larger collections
- use place value to work with patterns on a hundreds number chart.

Materials needed:

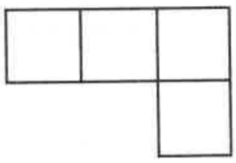
- base 10 materials
- a hundreds chart
- number jigsaw pieces.

'Good' questions and Teacher notes

1 **What numbers can you make that are below 100 and have 6 in the tens place?**
This will indicate if children understand that the value of a digit depends on its position within a number.

2 **I am thinking of a number between 10 and 100 with a single 9 in it. What might my number be?**
Note if children are able to find all possible answers. Ask them how they can tell if they have them all.

3 **You have a hundreds chart and jigsaw pieces. One of your pieces is shaped like this:**



If that shape covers the number 43, what other numbers might it be covering?
Most children will initially need to refer to the hundreds chart but encourage them to visualise as much as possible. This helps them to see the patterns that connect a particular number to its neighbours. There are 12 different answers to the question.

4 **What do you know and what can you find out about the number 180?**
Accept any suitable responses. Question further any children who do not include a response that shows their awareness of place value, e.g. 180 is 100 + 80, or 18 groups of 10, etc.

5 **What numbers can you make using 6, 5 and 8?**
Do children record their answers systematically and know when they have recorded all possibilities? Do any children include single digits, e.g. 5, as one of their answers?

6 **What numbers can be rounded off to 20?**

This will show you which children understand the concept of rounding off. Do they find all numbers, i.e. 15–24?

7 **How many three-digit number plates can you make or find with 3 in the tens place?**

If you have safe access to a car park you might let children use that to find any suitable number plates. It is still a good idea to have children write down some of their own.

8 **If Jane threw 3 tens and 5 ones to make 35, and Jim threw 4 tens and 2 ones to make 42, what tens and ones could you throw to win the game if to win you have to throw a number between 35 and 42?**

Make sure children write down the possibilities as tens and ones, not just the numbers between 35 and 42.

9 **How many different ways can you make the number 20 using only the numbers 10 and 1? You may use each of these numbers as many times as you wish in each sum or not at all.**

This question helps children to see that 20 can be named in many ways.

10 **Using base 10 materials how many ways can you model the number 25?**

It is important that children do not attach a set value to base 10 materials such as always calling the minis 1 or the longs 10. They should understand that if a flat is worth 10, then a long will be worth 1, etc.

11 **I wrote down a number with one zero in it, but I cannot remember what it was. I know it was between 500 and 800. What might it have been?**

Can children find all possible answers? How do they know they have found them all?

12 **A two-digit number contains exactly one 4. What might the number be?**

It is important to present the answers succinctly. How many different numbers might there be?

13 **I am thinking of a number between 10 and 70. Its tens digit is one more than its units digit. What might the number be?**

Note any children who do not understand the language of place value.

PLACE VALUE (Middle)

Experiences at this level will help children to:

- use place value to order and record up to five-digit numbers and decimals to two places
- round to 10, 100, 1000 or 10 000 for estimation
- extend multiplication facts using place value, e.g. 3 × 5 = 15, so 3 × 5 tens = 15 tens
- use place value to explain number patterns.

Materials needed:

- price catalogues
- base 10 materials
- a hundreds chart and a chart with numbers from 0.1 to 12
- Montessori cards
- an abacus.

'Good' questions and Teacher notes

1 **A number has been rounded off to 1200. What might the number be?**
This depends on whether it has been rounded off to the nearest 10 or nearest 100. Numbers from 1195 to 1204 would round off to 1200 as the nearest 10, or numbers from 1150 to 1249 would round off to 1200 as the nearest 100.

2 **How many numbers can you write with 8 in the hundreds place?**
Note if children only write numbers starting with 800 or if they write numbers above 1000.

3 **How many numbers can you make using the digits 1, 2, 3 and 4? You can only use each digit once in each number.**
Do children record their answers systematically and know when they have recorded all possibilities?

4 **Write as many numbers as you can that have a 7 in the tenths place.**
Note if children record numbers that only have tenths or if they record numbers that have hundredths too, for example, 2.7 and 2.79

5 **How many articles/items can you find in a catalogue with prices that have a 1 in the units place and a 9 in the tenths place?**
To do this children have to be able to recognise the units and tenths places within larger numbers, e.g. $21.95, $1.90, etc.

6 **We are numbers that look like □.□ and we are between 3.0 and 8.0. One of our digits is 6. What numbers might we be?**
Do children record their answers systematically?

7 **We are numbers that look like □□.□ and we are between 12.0 and 29.0. Our tenths digit is more than our units digit. What numbers might we be?**
Do children recognise the value of each place? Do they record their answers systematically?

8 **How many ways can you rename 1265?**

How confident are children when moving between 1000s, 100s, 10s and ones? Two possible answers are 1000 + 200 + 60 + 5; 1000 + 100 + 150 + 15.

9 **Find some places where people such as your family use numbers. Look for numbers where zero is used but has no meaning.**

One example of this is the odometer on a car where zero appears in front of the total kilometres travelled.

10 **All the numbers from a hundreds chart fell off when I moved it. I picked up one number but I could not work out which was the top and which was the bottom. What might the number be?**

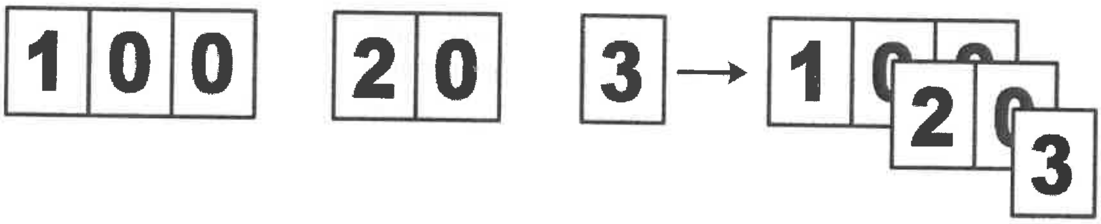
Do children use place value ideas to solve this one?

11 **Two numbers multiply to make 360. One of them has a zero on the end. What might the two numbers be?**

Can children find all possibilities? Can they see the pattern between pairs of numbers?

12 **I often use Montessori cards to make numbers. One day when I was just about to make some numbers I realised that I had lost all the tens. I still had the units and hundreds. What numbers could I still make?**

Montessori cards are sets of cards that fit together to make larger numbers, as shown in the diagram. It would be useful to have a set so children who have not used them before can do so to work out this question.



13 **I have represented a number on a three-prong abacus. If I add one more to my number I would need to take all the counters on one column off and put one counter on another column to replace the counters I took off. What number might be on the abacus and what might the new number be?**

Allow children to investigate this with an abacus if they need it. Are there any children who do not understand how an abacus works? Note those children who can generalise that any number with 9 in the tens or units will answer this question.

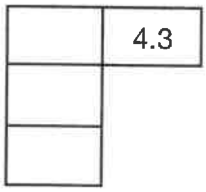
14 **An easy way to add 9 is to add 10 and take away 1. Using a similar strategy what other numbers might I add or subtract in this way?**

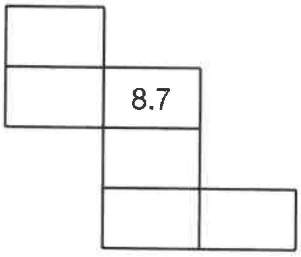
Children should be able to use other strategies such as to add 11, add 10 and then add 1 more; to subtract 99, take away 100 and add 1; etc.

15 One aspect of place value is visualising the patterns that connect a particular number to its neighbours. There is a range of activities that can involve a place value chart like this.

0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1
1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	2
2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9	3
3.1	3.2	3.3	3.4	3.5	3.6	3.7	3.8	3.9	4
4.1	4.2	4.3	4.4	4.5	4.6	4.7	4.8	4.9	5
5.1	5.2	5.3	5.4	5.5	5.6	5.7	5.8	5.9	6
6.1	6.2	6.3	6.4	6.5	6.6	6.7	6.8	6.9	7
7.1	7.2	7.3	7.4	7.5	7.6	7.7	7.8	7.9	8
8.1	8.2	8.3	8.4	8.5	8.6	8.7	8.8	8.9	9
9.1	9.2	9.3	9.4	9.5	9.6	9.7	9.8	9.9	10
10.1	10.2	10.3	10.4	10.5	10.6	10.7	10.8	10.9	11
11.1	11.2	11.3	11.4	11.5	11.6	11.7	11.8	11.9	12

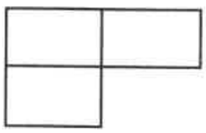
For example: You could cut the chart up to make a jigsaw. You could prepare a similar chart but with numbers missing. You could use jigsaw pieces with only one number shown for the students to complete the missing numbers, for example:

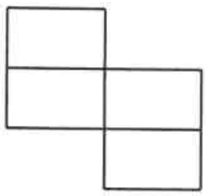




This then leads on to possible open-ended questions like:

The number 4.3 is covered by shapes like these:





What other numbers might be covered?

The main point of this activity is visualisation. Although some children will initially need to refer to the chart, encourage them to try to visualise as much as possible so that they improve their understanding of where numbers fit in our number system,

PLACE VALUE (Upper)

Experiences at this level will help children to:

- use place value to compare and order large numbers and decimals to thousandths
- round numbers for estimation purposes.

Materials needed:

- materials to model numbers
- sets of cards: [0], [0], [1], [1], [2], [2]

'Good' questions and Teacher notes

1 **What numbers can you make using 1, 0, 2, 7, 8 and 4?**

Children should record their answers methodically. You could ask them to write down the largest or smallest number it is possible to make using all the digits.

2 **Our flight was approximately 2000 km long. What might the actual distance be?**

This can be anything from 1950 to 2049 km. Most children can give one number that can be rounded. If they can give the entire range it shows they have a complete understanding of the concept of rounding.

3 **Represent 247 in as many different ways as you can.**

Variety is important here. Have materials available for children to use. Do they use systems that take advantage of place value?

4 **Someone has written a number in words, but the words have been cut up. The words and symbols used are:**

[three] [four] [sixty] [seven] [and] [,] [hundred] [.]
[thousand] [eight]

What might the original number be?

Children should be able to give a variety of answers. When they have some answers ask them if any of the words have to go in the same place each time.

5 **Make a set of cards. Each student should have access to cards that show the following numbers: 0, 0, 1, 1, 2, 2**

Ask students to show how these cards can be placed to make the following sentence true:
□.□ < □.□

Do students understand the function of the decimal point? Do they realise that the tenths digit on the left of the sentence does not have to be smaller than the tenths digit on the right of the sentence and that this is dependent on the size of the units digit. For example, $1.2 < 2.1$ but 2.1 is not less than 1.2

6 **Two numbers multiply to give 36 000. What might the two numbers be?**

Yes, this is a place value question! The key aspect is the zeros. Note how children handle them.

7 **Write a number larger than one million. Write a number larger than ten million. How do you know the first number is larger than one million and the second larger than ten million?**
Children should use place value concepts to explain their choice of numbers.

8 **Write a seven- or eight- digit number on the board. Ask children to write a number that is larger and a number that is smaller. Both must have the same number of digits as the number you wrote on the board.**
Record many of the children's numbers on the board and discuss how they know each number is larger or smaller than the given number. Look for explanations that involve the value of the places in the numbers.

COUNTING, PATTERN and ORDER (Junior)

Experiences at this level will help children to:

- compare and order using one-to-one correspondence
- write, say and count numbers to 10 and beyond
- count forwards and backwards (ones to fives to 100, and 10s and 100s to 1000)
- use ordinal numbers up to 10
- recognise odd and even numbers
- recognise, make and continue various patterns.

Materials needed:

- number charts
- number cards
- calculators.

'Good' questions and Teacher notes

1 **How many different counting patterns can you make about the number 3?**
Note how well children can tell you about their patterns. Do they count forwards and backwards? Is anyone able to state that when counting by threes you get an odd, even, odd, even pattern?

2 **Lan's group made a pattern with one person sitting, then one person standing, one sitting, one standing ... What patterns can you make with eight people?**
Children could work in a group of eight to do this. Allow time to show others some of the patterns.

3 **A train has some carriages. Each carriage has some dogs in it. Draw the train so that the order of the carriages from the front is different from the number of dogs in it (i.e. the second carriage cannot have two dogs).**
This question allows children to see that ordinal numbers are not the same as counting numbers. Check that children can distinguish between order and counting numbers.

4 **Write down everything you can about the number 12.**
Not only can you see what the children know but they can become aware of what they themselves know. Repeat for other numbers.

5 **Hold up a card with a number written on it, for example, 12. Ask children to record (or say) a number less than 12, and a number more than 12.**
This activity can be used for a range of numbers.

6 **Hold up two cards each with a number written on it, for example, 15 and 24. Ask children to record (or say) some numbers that come between these two numbers.**
The numbers can be changed to suit the group of children. You could simply record two numbers on the board with three spaces in between and ask children to write in order three numbers that come between the numbers shown.

7 **Design a set of numerals for a calculator which uses only straight lines.**
The numerals should be unique and recognisable.

8 **Write down some odd numbers between 0 and 100.**
Check that all numbers the children write are odd. Do they record them haphazardly or methodically?

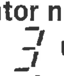
9 **I have written a secret number between 50 and 70. It is an even number. What might it be?**
Can children write them all? Repeat for other numbers.

10 **I have written a secret number that is more than 65. What might it be?**
Extend this by asking children to write the biggest number they can. Compare these numbers to see which is the biggest. Repeat for other numbers.

11 **Place some number cards on a ledge not in order. Ask children to take different cards, for example, a card with a number larger than 18, a number smaller than 13, a number between 20 and 30 …**
This activity can be adapted to suit the children taking part.

12 **Create a number pattern where no consecutive numbers differ by 1 or 5.**
Discuss what this means as some children might find it difficult to understand. They might like to check each other's finished patterns. Can they describe their pattern in words?

13 **Create some number patterns where the difference between the numbers is 10.**
Can children describe their pattern pointing out features such as, if it goes forwards or backwards, what the starting point is and the size of the numbers?

14 **Using calculator numbers I want to design a three-digit number for my number plate that uses 12 lines, e.g.**  **uses 5 lines.**
Children can check each other's responses. Are they able to make a number of different responses?

15 **Make a counting backwards pattern, starting at 105.**
Children should describe their pattern. Note how sophisticated the patterns are.

16 **I am thinking of a number. It is not spelt the way it sounds. What might my number be?**
A class discussion of what the children find would be useful. Some possible answers are one, two and eight.

17 **What numbers are in the pattern 2, 4, 6, 8, ... and also in the pattern 5, 10, 15, 20 ...?**
Do children realise that all multiples of 10 will be in both patterns because they are even numbers and that multiples of 5 will not be in the 2's pattern because they are odd numbers? Can any children take this pattern beyond the tables range?

COUNTING, PATTERN AND ORDER (Middle)

Experiences at this level will help children to:

- read, write and say whole numbers
- count forwards and backwards by numbers to 10 beyond 100, starting from any number
- use materials to produce number sequences, e.g. square numbers
- produce and describe number patterns
- recognise patterns in multiplication tables.

Materials needed:

- number charts
- calculators (useful for checking or producing number patterns).

'Good' questions and Teacher notes

1 **Which number in this group does not belong? Why?**

15 2 8 13 16

Children must explain their responses. Their answers can differ. Are their explanations coherent?

2 **What do you know and what can you find out about the multiples of 3, i.e. 3, 6, 9, 12, 15, 18, 21 ...?**
One response could be that the sums of the digits of the multiples of 3 are multiples of 3.

3 **Create a pattern starting at 91 that someone else can continue.**
It is important that children can describe their pattern to others. They could check each other's.

4 **Make up a four-digit PIN number and say why it is easy to remember.**
The emphasis here is on being able to communicate the reason to someone else. You could possibly make restrictions such as not using birth years.

5 **Show these numbers on the given number lines:**

10 and 60 ______________________________

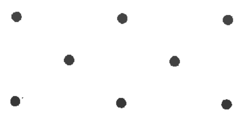
10, 60 and 90 ______________________________

Write 5 other numbers on your number lines.
Note if students realise that the largest number does not have to be put at the very end of the line. The placement of each number is dependent on the other number/s.

6 **I doubled a number and kept doubling so that the original number was doubled four times. What might the answer be?**
The answer will be a multiple of 16 (16, 32, 48…). Do children use all the information in the question?

7 **I halved a number and kept halving for a total of five times. The answer was a whole number. What number might I have started with?**
It will be an even number and a multiple of 32. Ask children to describe how they found an answer.

8 **A number was shown as a set of dots. Part of the pattern looks like**



What might the number be? How do you know?
Accept any reasonable answers as long as children can justify them.

9 **How might you use each digit 7 2 3 6 4 once and only once to make both of these sentences true at the same time: ❒ > ❒.❒ and ❒ < ❒ ?**
Note how many different answers children can find. Are they confident with the symbols used?

10 **The number 8 is half way between two numbers. What might those two numbers be?**
How do children approach this one? The obvious answer is 7 and 9, but can children use this to derive other pairs of numbers such as 6 and 10, 5 and 11, 4 and 12 …?

11 **My father is double my age. How old might I be?**

12 **My dog is half as old as me. How old might I be and how old is the dog?**

13 **My dog is half as old as me. My mother is double my age. How old might we each be?**
Q11, Q12 & Q13 For these three questions the ages should be within a practical range. For example, in Q13 ages of 2 (dog), 4 (me), and 8 (my mother) or 4 (dog), 8 (me), and 16 (my mother) are not acceptable. Note if children develop systems based on the inherent patterns to find responses.

COUNTING, PATTERN AND ORDER (Upper)

Experiences at this level will help children to:

- count beyond 1000 starting from any number
- continue, produce and describe sequences of fractions and decimals
- continue, produce and describe sequences involving constant multiplication and division or combinations of operations
- identify and work with prime numbers.

Materials needed:

- number charts
- calculators (useful for checking or producing number patterns).

'Good' questions and Teacher notes

1 **To find the value of a word if A = 1, B = 2, C = 3, etc. add the value of each letter in the word. What are some words which are worth between 35 and 40? What is a sentence worth more than 500?**

When children have done this they could invent their own letter values to work out other words or sentences.

2 **Write ten numbers that come between 699 995 and 700 020.**

Adapt the numbers to suit your class.

3 **Three consecutive even numbers add to give a number between 100 and 120. What might the numbers be?**

Note how concisely children can communicate their answers. It is interesting to listen to them tell how they got their answers.

4 **I am thinking of a number. The hundreds digit is larger than the units digit. The tens digit is larger than the hundreds and it is odd. What might my number be?**

Do children use all of the information to get an answer? Note if they check their answer against the question.

5 **Ask children to make up a set of five clues about a number. It must be possible to work out the number after the five clues have been given.**

An example for the number 23 might be: 1) It is an odd number 2) It has two digits 3) It is a prime number 4) It is below 40 5) Its digits add to 5.

Let other children work out the numbers. The clues children make up tell you lots about what they know about numbers.

6 **Create a pattern starting at 2.05 that someone else can continue.**

Are children able to describe their pattern? How sophisticated are the responses? Let them check each other's patterns.

7 **Show these numbers on the given number line:**

0.1 and 1

Write 5 other numbers on your number line.

Do students think that they have to place a zero at the start of the line or even that 0.1 has to be placed at the start? Do they think that 1 has to be placed at the end of the line? You could place 0.1 and 1 somewhere along the line and ask students to show the position of some numbers that could come before 0.1, between 0.1 and 1, and after 1.

8 **I am thinking of a number. If I divide by 3 there is a remainder of 1. If I divide by 4 there is a remainder of 1. What might my number be?**

The pattern is important in this question. Are children able to describe it?

9 **I used a scientific calculator to work out a number sentence and got the answer 20. I know there were two sets of brackets, a division sign and a subtraction sign but I cannot remember any of the other signs or the numbers. What might the number sentence be?**

One possible solution is $(5 \times 5) - (5 \div 1)$

10 **What numbers have factors of both 5 and 6?**

Do children know what a factor is? Can they work this out systematically and use the pattern to predict large numbers?

11 **When I count by twos I land on both 100 and 1000. When I count by threes I do not land on either. What can you count by so that you don't land on 100 but you do land on 1000? What can you count by to land on both 100 and 1000?**

Note the method children use to investigate these patterns.

OPERATIONS (Junior)

Experiences at this level will help children to:

- use materials to develop number bonds
- learn about and use doubles, addition and subtraction facts to at least 20
- add and subtract small numbers in story problems
- model word problems
- add and subtract with trading using materials
- use materials to show grouping and division
- write number sentences and make up stories about number sentences.

Materials needed:

- counters and sticks
- calculators
- number lines drawn on card strips
- toy vehicles (with different numbers of wheels)
- division 'mats'
- a collection of pictures of groups and arrays.

'Good' questions and Teacher notes

1 **Give children five objects. Tell them to pretend that the five objects belonged to two people, some to one person and some to the other person. Work out how many might have belonged to each person.**

This question can be repeated for other numbers of objects.

2 **Ask children to make up a problem of their own that they are able to do in their head. Make time to share the problems and the ways that children worked them out.**

This activity will tell you a lot about the strategies children are using.

3 **A basketballer scored 9 points in two games. What might her scores in each of the games be?**

4 **❒ + ❒ + ❒ = 13 What might the missing numbers be?**

Q3 & Q4 The purpose of these questions is for children to think about the addition process in a different way and to show that there can be a range of possible answers to a given problem.

5 **This room has three chairs more than the room next door. How many chairs might be in each room?**

Do children realise that there is more than one possible answer? Have some counters available for children to model the situation. Do they find a solution by adding on or by counting back? After children have done this one it is a good idea to discuss and model the ways different children worked it out.

6 **The Melbourne Tigers are leading the Canberra Cannons by 4 points. What might the scoreboard look like?**

This is similar to the previous question. Note if children count on or count back. Can they give more than one solution?

7 **There are now only four chickens in Mrs Farmer's pen. How many chickens did she once have, and what happened to them?**

This is a different focus for subtraction. Note the children who use larger numbers confidently. It is important to use counters or similar aids.

8 **Give children a number line. Ask them to work out how to use the number line to help them add two numbers together.**

It would be helpful to have some number lines drawn on strips of card so children can touch the numbers as they are working. You need at least one number line per three children. Choose numbers in a range suitable for the children with whom you are working. Note if children start at the larger number and count on the smaller number.

9 **Work out how to use a number line to find the answer to 12 – 8. Work out how to use the same number line to find the difference between 8 and 12.**

Again it is useful to have number lines drawn on card strips for children to work with. The purpose of this question is to show that the difference between two numbers can be worked out by counting back or by counting up from one number to another.

10 **Make up some different ways to add 5 to 8 in your head. In how many ways can you do it?**

This is to show that there is not only one correct way to do a calculation. Listen to all responses and note the children who have not developed 10 as a bridging number, e.g. add 2 to 8 to make 10 and then add the other 3 to make 13, or 5 and 5 make 10 and 3 more make 13.

11 **The difference between two numbers is 5. What might the two numbers be?**

This focuses on 'difference' as subtraction. Some children will feel confident using larger numbers. Some may record their answers systematically, e.g. 6 – 1, 7 – 2, 8 – 3, 9 – 4, etc.

12 **Choose a number between 5 and 20. Children are to write all the subtractions they can using that number. For example, if the number was 11 they could write 11 – 6 = 5; 15 – 11 = 4; 13 – 2 = 11 ...**

Note those children who can generate strings of related facts and use the number in different positions in their subtractions, that is, the starting number, the amount taken away or the answer.

13 **How can you make up a sentence with five words but no more than 20 letters?**

At this stage children will most likely use the total of 20 to work this out rather than balancing the number of letters above and below 4. In finding a response children will use both addition and subtraction.

14 The faces of this cube are numbered consecutively. What might the sum of the faces be?

The faces we cannot see could be numbered 10, 11 and 12 or 4, 5 and 6, or in between, so the sum could be 39, 45, 51 or 57. Let children use a calculator to add the numbers. You will need to discuss what 'consecutively' means.

15 Write the number 19 on the board and tell children that it is the answer to some number sentences. Explain that they are allowed to use any of the following calculator keys: [+] [−] [=] and any of the digit keys to find number sentences that equal 19. They record their findings.
Note if children use + and − in the same number sentence or if they use combinations of more than two numbers, for example, 6 + 10 + 3 = 19 or 25 − 3 − 3 = 19.

16 Yesterday I put some counters into groups with the same number in each group. I cannot remember the groups, but I can remember that there were 12 counters. What might the groups have been?
Children need counters to do this. The possibilities are two groups of 6, three groups of 4, four groups of 3, or six groups of 2.

17 a) Twelve teddies all sat down at a café which had 4 tables. Show how the teddies might have sat at each table.
Groups of children will need 12 teddies, or counters to represent them, and a mat with 4 square 'tables'. Children should show some different possibilities. Who has the same number of teddies at each table?

b) Twelve teddies all sat down at a café which had 5 tables. Show how the teddies might have sat at each table.
For this question children will need 12 teddies and a mat with 5 square 'tables'. Do children realise that with 5 tables they cannot have the same number of teddies at each table?

18 The chef has 24 pizzas on trays in the oven. There is the same number of pizzas on each tray. What might this look like?
Allow children to use counters to help with this. The possibilities are 2 trays of 12, 3 trays of 8, 4 trays of 6, 6 trays of 4, 8 trays of 3, and 12 trays of 2.

19 What things do you know that come in twos (or pairs)?
Either write or draw what the children suggest. To extend children's thinking pose problems like 'Using these sticks, show me what 5 pairs of chopsticks would look like'. You could also ask children to say what things they know that come in sets, bunches or groups.

20 There are five vehicles in the car park. How many wheels might there be?
Let children look at toy vehicles with different numbers of wheels. Do children add numbers of wheels separately or do they use doubles or other suitable strategies?
You could look at situations where the number of wheels might be an odd number.

21 **Eighteen people said they wanted to do folk dances. The teacher said they must dance in groups, but no one must be left out. How many different types of groups can you make?**
This is the same principle as Q16. Let children use counters to represent the dancers.

22 **When the children in a class got into groups of four there was one child left over. How many children might there be in the class?**
This would be good to model with the class. Can anyone do this by multiplying by 4 and adding 1?

23 **Write a number story about the picture.**
Have a collection of pictures of groups and arrays and choose one for children to look at. It is important that children recognise that there is more than one possible story.

OPERATIONS (Middle)

Experiences at this level will help children to:

- practise multiplication facts up to and including the 10 times tables and relate these to division
- build on known number facts, and use extended number facts
- double and halve
- develop mental strategies and estimation skills
- refine methods for addition and subtraction
- record simple multiplication and division calculations
- select the appropriate operation to solve whole number problems.

Materials needed:

- counters
- base 10 materials
- calculators
- Unifix cubes.

'Good' questions and Teacher notes

1 **Make up some different ways to add 9 to 23 in your head. In how many ways can you do it?**
This is to show that there is not only one correct way to do a calculation. Listen to all responses and note the children who have not developed multiples of 10 as bridging numbers, e.g. add 7 to 23 to make 30 and then add the other 2 to make 32. Who does it by adding 10 and then subtracting 1?

2 **Five numbers added together make an odd number. What do you know about the numbers?**
Either one, three or five of the numbers must be odd. This question highlights some features of odd and even numbers.

3 **I subtracted an odd number from an even number and got the answer of 41. What might the odd and even numbers be?**
Do children record their number sentences systematically? Can they describe the pattern they find? This task could lead to an investigation of the result of adding or subtracting two odd numbers or two even numbers or one odd number and one even number.

4 3☐ + 1☐ = ☐2

What might the missing numbers be?

This is to show that there are many possible answers. You could ask children to find all possibilities. Note those children who have difficulty working out solutions with trading.

5 **Last night I added together two numbers, each with two digits. I got an answer of 67 but I cannot remember what the numbers were. Help me work out some possibilities.**

Do children approach this systematically? The question is similar to the previous one but is stated in a more abstract manner.

6 **Using only your head, work out what the missing numbers might be: 2☐ + 3☐ = ☐0**

The purpose of this question is to see if children understand that the two units digits must either be zero or have a combined total of 10, and that when they have a total of 10, the answer will be 60.

7 **Using only your head, work out what the missing numbers might be: 5☐ – 3☐ = ☐0**

This question is similar to the previous one.

8 **Uncle Norm cannot subtract numbers in his head. He does not have a pen and paper but he does have a calculator. Unfortunately the 5 and 7 buttons are broken. How could Uncle Norm use the calculator to find 75 – 56?**

This question highlights the equal addition principle for subtraction, e.g. add 5 to each number (80 – 61).

9 **I have some marbles. I give some away to my friends and am left with 15. How many marbles might I have started with and how many might I have given away?**

Again, look for a range of answers. Note the size of the numbers children are confident working with.

10 **I did a subtraction task and the answer was 215 but I cannot remember the other numbers. Find as many solutions to this subtraction as possible.**

Which children develop a system? Note the numbers children use to do this task as they will tell you a lot about the stages children are at. For example, do they record subtractions like 692 – 477? When children are working out the numbers do they use addition to find the result and then use the number and the result to record a subtraction?

11 **A number is divided by 5 and leaves a remainder of 3. What might the number be?**

Do children do this by trial and error or do they develop a rule, i.e. multiply any number by 5 and add 3?

12 **Your friend Sally is trying to work out how to double numbers in her head. When she was asked to double 34 she said 68. When she was asked to double 23 she said 46. When she was asked to double 27 she said 44. When she was asked to double 36 she said 62. What are some other double questions that Sally might get wrong? What might be your advice to your friend?**

The advice could relate to exchanging ten units for one ten when the units digit that is doubled results in an answer of 10 or more, and adding it to the other tens.

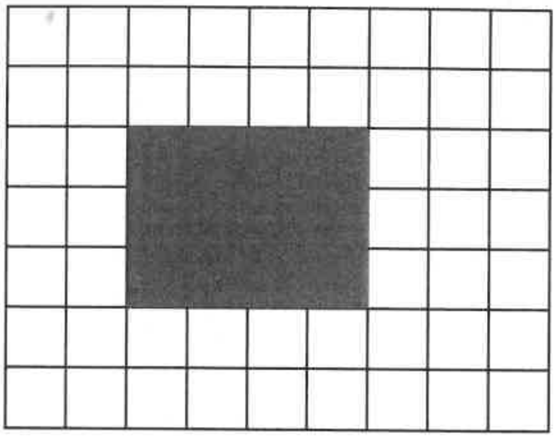
13 Make up a sentence with each word having the same number of letters. How many letters are there altogether?

The purpose of this question is for children to use multiplication to calculate the number of letters.

14 At a party the lollies were shared. Each person got 3. How many people were at the party and how many lollies were there altogether?

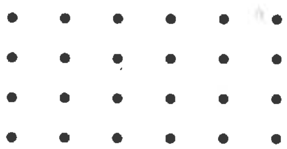
Note how children do this. Do they realise that they only have to multiply 3 by any number of people to find the total number of lollies at the party? For example, 3 (lollies) x 9 (people) = 27 (lollies).

15 How many squares are covered?



Note how children work this one out. Do they draw in the squares and count them or do they use multiplication to work out the number covered? You could extend this by asking children to find the number of squares that are not covered. To do so they will have to find the total number of squares and subtract the number of squares that are covered, i.e. 63 – 12 = 51.

16 Write as many number facts from this array as possible.



Note if children recognise the relationship between multiplication and division.

17 What might the missing numbers be? ❒ × ❒ = 36

Children should be able to find all whole number possibilities. Do they know that 3 × 12 is the same as 12 × 3, 4 × 9 the same as 9 × 4 …? Some children might like to use fractions in their answers, for example, $\frac{1}{2} \times 72 = 36$.

18 What might the missing numbers be? ❒❒ ÷ ❒ = 3

There are 6 possibilities: 12 ÷ 4, 15 ÷ 5, 18 ÷ 6, 21 ÷ 7, 24 ÷ 8 and 27 ÷ 9. Do children use their knowledge of the 3 times table to help them?

19 **I have some Unifix cubes in my pocket and I notice that when I share them into two equal groups there is one left over, but when I share them into three equal groups there are two left over. How many might there be in my pocket?**

Make Unifix available for children to work with. Do children realise that they must start with an odd number of cubes? Ask children to develop a general rule for this.

20 **What are some answers you can make to multiplication questions using the numbers 3, 4 and 5 once each?**

An example of a multiplication using each number once is 53 × 4. You may prefer to let children use calculators to do this.

OPERATIONS (Upper)

Experiences at this level will help children to:

- recall multiplication and division facts up to and including 10 × 10
- choose and use mental strategies
- estimate and check reasonableness of answers
- refine methods for addition and subtraction of whole numbers and decimals
- use suitable written methods for multiplication and division
- select the appropriate operation to solve number problems.

Materials needed:

- calculators
- concrete materials where necessary.

'Good' questions and Teacher notes

1 **The answer to a division question is 5. What might the question be?**

Note how children do this. Do they know that if they multiply 5 by any number they will find the numbers to make their question, e.g. 5 × 20 = 100, so 100 ÷ 20 = 5?

2 **2□6 + □8 = □2□**

Work out all possible answers for this addition.

There are nine possible answers and 10 if you allow a zero to be used before the 8.

3 **Using all of the digits 9, 8, 7, 6, 5, 4, 3, 2, 1 and any operations, what numbers can you make?**

Children could also be asked to find the largest or smallest number they can make.

4 **What might the missing numbers be?**

```
  □□□□
+ □□□□
  ----
  5000
```

Do students realise that there are many possible solutions? Can they find the range of possible solutions?

5 **I did a subtraction task last night but can only remember the answer and that it looked like this:**

```
  ❐❐❐
– ❐❐❐
─────
   5 7
```

What might the missing numbers be? Try to describe all possible answers.

This question focuses on the difference between numbers. The smallest possible top number is 157 – 100 = 57 and the largest is 999 – 942 = 57. There are 843 possible combinations.

6 **5❐2 – ❐❐4 = 68 What might the missing numbers be?**

There are ten possible answers.

7 **A school has 400 students. They all come to school by bus, and each bus carries the same number of students. How many students might there be on each bus?**

Note the methods children use to do this. Do they use division or multiplication? Do they use extended number facts?

8 **Two problems for 8 ÷ 2 are:**

There are 8 tennis players. How many teams of 2 are there? and

There are 8 tennis players. If you make 2 teams, how many players will be in each team?

Make up two different types of word problem for 36 ÷ 3

Make sure to share the word problems as a class and discuss how they differ. The first problem given is an example of division as 'how many groups' (quotition) and the second problem is an example of division as 'sharing' (partition).

9 **❐0 ÷ ❐ = ❐ What might the missing numbers be?**

There are 8 possibilities: 10 ÷ 2 = 5, 10 ÷ 5 = 2, 20 ÷ 4 = 5, 20 ÷ 5 = 4, 30 ÷ 5 = 6, 30 ÷ 6 = 5, 40 ÷ 5 = 8 and 40 ÷ 8 = 5

10 **(a) Using 9, 8, and 7 set out like ❐ ❐ × ❐ how many different answers can you get?**

(b) Using 9, 8, 7 and 0 set out like ❐ ❐ × ❐ ❐ how many different answers can you get?

(c) Using 9, 8, 7 and 0 set out like ❐ ❐ ❐ × ❐ how many different answers can you get?

Note the children who do this systematically.

11 *** × • = 2280**

What might * and • be? How many different answers can you find?

Again, which children do this by division and which do it by trial and error multiplication? Do they use estimation skills?

12 **(a) How could you calculate 23 × 4 if the 4 button on your calculator is broken?**

(b) How could you calculate 23 × 21 if the 2 button on your calculator is broken?

For (a) the children could press 23 + ==== or 23+23+23+23=. For (b) they could press 13 + === (21 times), then add 10 (21 times).

This is quite a difficult task but it provides a rich investigation.

13 How could you calculate 17 x 26 if the 7 button on your calculator is broken?

Look for answers that show children realise that 17 can be partitioned in ways other than 10 & 7, for example, 8 & 9. Also look for answers where children multiply 26 by another number such as 18, e.g. 18×26 and then subtract 26, or 16×26 and then add 26.

14 Eighty-four children in four grades are arranged into teams with the same number in each team. How many teams are there and how many children might there be in each team?

As for Q7, note the methods children use to do this. Do they use division or multiplication? Do they use known number facts?

15 2▢ x 3▢= ▢▢0 What might the missing numbers be?

Allow children to use calculators for this one. Do children use the tables to help them? For example, $5 \times 6 = 30$ so the answer to 25×36 will end in zero, and $2 \times 5 = 10$ so $22 \times 35 = 770$ and $25 \times 32 = 800$ …

16 Write a word problem where the answer is $27\frac{1}{4}$. Write a word problem where the answer is 27 and 1 remainder.

Are children able to deal with remainders in a sensible manner and know when to convert them to fractions and when to leave them as remainders? For example, if you share 109 meat pies among four families each family would get 27 whole pies and the remaining one could be cut into quarters. If you are putting 109 people on four buses you would put 27 people on each bus and fit the remaining person on one of the buses.

17 Work out a way to multiply a number by 99.

Although the most efficient method is probably to multiply by 100 and then subtract the number, for example $15 \times 99 = 15 \times 100 - 15$, it is not the only way. Share the methods that children find. You could provide calculators so children can check if their method works for other numbers.

18 What could you add to 361 to make it divisible by 10?

Adding 9 is the obvious answer, but there are many other possible solutions. Can children find a rule for this one?

19 Using four 4s and any operation, how many different answers can you make?

It is possible to make all numbers between 1 and 100!

20 What might the missing numbers be? ▢▢÷ 5 = ▢▢

There are 10 possibilities: $50 \div 5 = 10$, $55 \div 5 = 11$, $60 \div 5 = 12$, $65 \div 5 = 13$ … $95 \div 5 = 19$. Do children do this systematically? Do they use multiplication to help them work out possible answers?

21 Suppose you have a broken calculator, and the only buttons that work are [3], [5], [0], [×] and [=]. What calculations can you work out the answer to? Write down each calculation and answer.

As well as simple calculations of the type $5 \times 3 =$, do children also include types like $35 \times 35 \times 35 =$, $530 \times 5 \times 3 =$, $333 \times 55 =$ or $5 \times 5 = = = =$ …?