



Rich Learning Tasks

Dr. Marian Small



Contents

Number

Planting Trees.....	1
Original Prices.....	4
Fitting In.....	7

Measurement

Combined Shape.....	10
Wrapping a Prism.....	13
Predicting Area.....	16

Patterns and Algebra

Equal for 10.....	19
Building a Mean	22

Name: _____

Number

Planting Trees

Kyla can plant about 300 trees in an hour.

Mia can plant a single tree in about 13 seconds.

If they work together, how long would it take them to plant 100 trees?

What's the point of this task?

In order to solve this problem, students are likely to work with equivalent (unit) rates. Knowing that Kyla can plant 300 trees in an hour also tells you she can plant 5 trees in a minute or that in 12 seconds she can plant one tree. Knowing that Mia can plant one tree in about 13 seconds means she can plant 4 trees in 52 seconds, so about $4\frac{1}{2}$ trees in a minute or about 270 trees in an hour. Then students need to think about how knowing how many trees the two, together, can plant in a minute or the number of seconds it takes each to plant a tree, helps them figure out how long it would take to plant 100 trees.

A number of the questions below focus on using the information given to estimate. Estimation should always be encouraged in ratio and rate problems.

Questions to facilitate the learning

- How could you have predicted it would take less than 20 minutes?
- Who is faster—Mia or Kyla? How do you know?
- Why did you have to change the description of at least one or both of the rates to help you?

Scaffolding the learning

- How long does it take Kyla to plant a tree? Is that information useful?
- How many trees can Mia plant in an hour? Is that information useful?

Extending the learning

Students might choose different speeds for Kyla and Mia that would require them to need exactly 12 minutes to plant 100 trees.

Rubric

Level 1	Level 2	Level 3	Level 4
<p>The student has no idea of how to determine the solution or just says $\frac{1}{3}$ hour by dividing 300 by 3.</p>	<p>The student estimates that it would take about 10 minutes to plant the 100 trees by assuming the rate for Mia is similar to the rate for Kyla.</p>	<p>The student calculates that it would take about 10.5 minutes to plant the 100 trees.</p> <p>The student can explain how he/she determined the solution.</p> <p>The student figures out that Kyla is faster than Mia, but it was not obvious to him/her.</p> <p>The student uses an alternate description of one or both of the given rates to solve the problem.</p>	<p>The student calculates that it would take about 10.5 minutes to plant the 100 trees.</p> <p>The student can clearly explain how he/she determined the solution.</p> <p>The student can easily explain why Kyla is faster than Mia, but not a lot faster.</p> <p>The student can clearly explain why different descriptions of one or both of the given rates were useful for solving the problem.</p>

Name: _____

Original Prices

30% of Number A is the same as 40% of Number B.

What could A and B be?

Think of lots of answers if you can.

What's the point of this task?

Rather than asking a student to simply calculate a percent, this problem is built on relationships. For example, we could have asked: If A is 50% of B, what percent of B is $\frac{1}{2}$ A? Instead, this problem is just slightly more complex.

Some students can solve the problem by just trying lots of numbers and looking for possibilities. For example, they might notice that 30% of 40 is the same as 40% of 30 or they might notice that 30% of 100 is the same as 40% of 75.

Questions to facilitate the learning

- Which number, A or B, has to be greater? Why?
- Could either number be the double of the other? Why or why not?
- How could thinking about using decimals to calculate a percent help you figure out one answer?
- How can you easily get another pair of numbers once you have a pair that works?

Scaffolding the learning

- How can you figure out 30% of a number?
- What percent of Number B is 15% of Number A? Why?
- What percent of Number A is 10% of Number B? Why?

Extending the learning

Suppose 60% of Number A is the same as 50% of Number B. How are A and B related?

Rubric

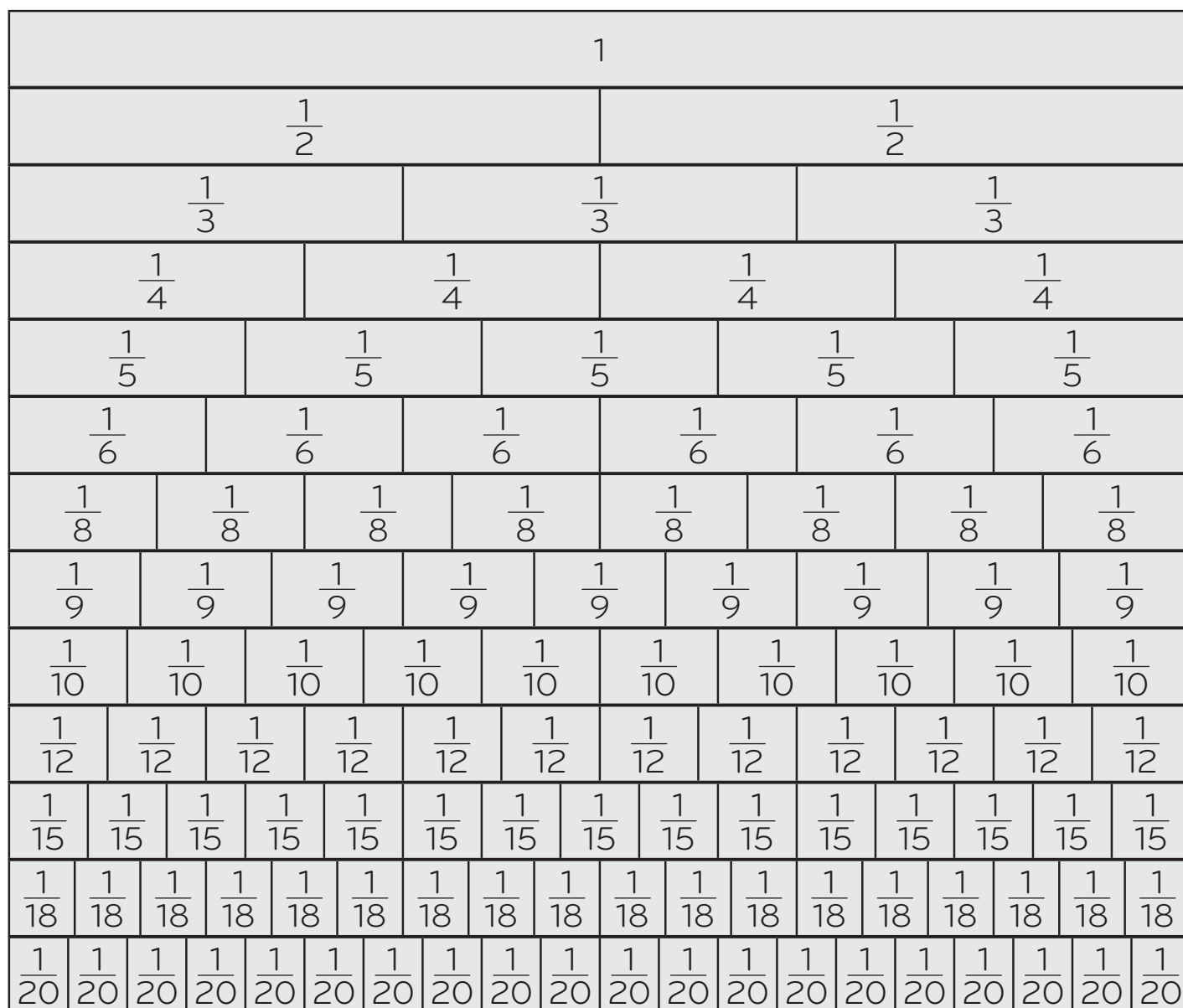
Level 1	Level 2	Level 3	Level 4
<p>The student is unable to determine even one pair of numbers that work.</p> <p>The student does not realise, in advance, that B must be less than A.</p> <p>The student does not know how to use one solution to easily get another.</p>	<p>The student determines at least one or two pairs of numbers that work. He or she can show that the calculations are correct.</p> <p>The student predicts that B must be less than A without giving clear reasons.</p> <p>The student does not know how to use one solution to easily get another.</p>	<p>The student determines at least three or four pairs of numbers that work. He or she can show that the calculations are correct.</p> <p>The student predicts that B must be less than A and gives some reasons.</p> <p>The student does not know how to use one solution to easily get another.</p>	<p>The student determines that, in general, B is 75% of A and clearly explains why.</p> <p>The student easily predicts and explains why B must be less than A without actually performing any calculations.</p> <p>The student shows an understanding that once A and B work, so does any pair created by multiplying A and B by the same amount.</p>

Fitting In

Look at the fraction tower.

What fraction fits into another fraction about $2\frac{1}{2}$ times?

Look for lots of possibilities.



What's the point of this task?

Asking students to find a fraction that fits into another exactly $2\frac{1}{2}$ times is actually asking a division or multiplication question, i.e. If $a \div b = 2\frac{1}{2}$, what are a and b ? or if $2\frac{1}{2} \times a = b$, what are a and b ? But using the fraction tower makes the task accessible even to those students whose skills with multiplying and dividing fractions are either missing or minimal. Those students can count on visual cues to help them.

Some students might use equivalent fractions to determine solutions. For example, since $\frac{2}{10} = \frac{1}{5}$ and $\frac{3}{15} = \frac{1}{5}$, that means $\frac{1}{10}$ fits into $\frac{1}{5}$ twice and $\frac{1}{15}$ fits into $\frac{1}{5}$ three times, so perhaps $\frac{1}{12}$ is a reasonable solution.

Although some students might use the tower to notice that $\frac{1}{5}$ and $\frac{1}{2}$ work, that $\frac{1}{10}$ and $\frac{1}{4}$ work, that $\frac{1}{15}$ and $\frac{1}{6}$ work and that $\frac{1}{20}$ and $\frac{1}{8}$ work, some students will generalise and realise that any fractions of the form $\frac{1}{5}n$ and $\frac{1}{2}n$ work, even though they do not see them on the tower. Other students will notice that if $\frac{1}{10}$ and $\frac{1}{4}$ work, so do $\frac{2}{10}$ and $\frac{2}{4}$ or $\frac{3}{10}$ and $\frac{3}{4}$, etc.

The use of the term 'about' leaves latitude for students to determine other solutions, e.g. $\frac{1}{9}$ and $\frac{1}{4}$ or $\frac{1}{20}$ and $\frac{1}{9}$.

Questions to facilitate the learning

- Could the two fractions be a fraction at the bottom of the tower and one near the top? Why or why not?
- How could using equivalent fractions help you figure out an answer?
- How could using fraction operations help you figure out an answer?

Scaffolding the learning

- How many $\frac{1}{4}$ s fit into $\frac{1}{2}$? How might that help you get a solution?
- What fraction fits into $\frac{1}{3}$ twice? Three times? How might that help you get a solution?

Extending the learning

Students might look for fractions that fit into other fractions about $1\frac{1}{3}$ times.

Rubric

Level 1	Level 2	Level 3	Level 4
<p>The student cannot determine even one pair of fractions where the smaller fits into the greater between $2\frac{1}{4}$ and $2\frac{3}{4}$ times.</p> <p>The student does not realise that once a solution is determined, the numerator and denominator can be multiplied by the same amount to determine another solution.</p> <p>The student cannot explain how using equivalent fractions and/or fraction operations can help solve the problem.</p>	<p>The student determines at least one pair of fractions where the smaller fits into the greater between $2\frac{1}{4}$ and $2\frac{3}{4}$ times.</p> <p>The student can explain how he/she knows he/she is correct.</p> <p>The student does not realise that once a solution is determined, the numerator and denominator can be multiplied by the same amount to determine another solution.</p> <p>The student cannot explain how using equivalent fractions and/or fraction operations can help solve the problem.</p>	<p>The student determines at least two or three pairs of fractions where the smaller fits into the greater between $2\frac{1}{4}$ and $2\frac{3}{4}$ times.</p> <p>The student can explain how he/she knows he/she is correct.</p> <p>The student does not realise that once a solution is determined, the numerator and denominator can be multiplied by the same amount to determine another solution.</p> <p>The student finds it difficult to explain how using equivalent fractions and/or fraction operations can help solve the problem.</p>	<p>The student determines at least four or five pairs of fractions where the smaller fits into the greater between $2\frac{1}{4}$ and $2\frac{3}{4}$ times.</p> <p>The student can clearly explain how he/she knows he/she is correct.</p> <p>The student realises that once a solution is determined, the numerator and denominator can be multiplied by the same amount to determine another solution.</p> <p>The student clearly explains how using equivalent fractions and/or fraction operations can help solve the problem.</p>

Combined Shape

A shape made up of 2 trapezoids and a triangle has an area of 50 cm^2 . Sketch the shape, indicate the dimensions and the area of each piece and prove that the total area really is 50 cm^2 .

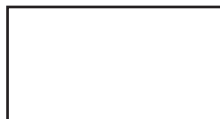
Look for different possibilities.

A large grid of dotted lines for sketching a combined shape. The grid is 15 units wide and 15 units high, forming a square area for drawing.

What's the point of this task?

This problem provides an opportunity to use formulas for areas of triangles, parallelograms and rectangles and perhaps trapezoids. Because they have the total area, instead of the area of each shape, students have much more latitude to create a solution.

Some students might start with a rectangle with an area of 50 cm^2 , separate it into two trapezoids and a triangle and indicate the dimensions of the pieces. They still need to describe the dimensions and areas of each piece.



Others will start with the trapezoids and triangle and use what they know about area formulas to ensure that the total area is correct. Students also have an opportunity to reinforce their understanding of what different trapezoids can look like.

Providing a grid in the background allows students to count squares if they need to.

Questions to facilitate the learning

- Do the trapezoids need to have the same area? Could they?
- Is it possible for all three shapes to have the same area? Explain.
- How do you know that there are many possible solutions?

Scaffolding the learning

- What can trapezoids look like?
- How do you figure out the area of a triangle?

Extending the learning

Students might create a shape made up of three or four of their own shape choices with a given total area.

Rubric

Level 1	Level 2	Level 3	Level 4
<p>The student cannot create the required shape.</p> <p>The student does not realise that there must be other solutions.</p> <p>The student does not necessarily realise that there are many choices for the proportions of the three subshapes of the final shape.</p>	<p>The student creates at least one shape made up of 2 trapezoids and a triangle with the total area of 50 cm^2.</p> <p>The student counts grid squares rather than using formulas to determine the areas of shapes.</p> <p>The student does not realise that there must be other solutions.</p> <p>The student does not necessarily realise that there are many choices for the proportions of the three subshapes of the final shape.</p>	<p>The student creates at least one shape made up of 2 trapezoids and a triangle with the total area of 50 cm^2.</p> <p>The student uses formulas to determine areas of shapes.</p> <p>The student struggles to explain why there must be other solutions.</p> <p>The student does not necessarily realise that there are many choices for the proportions of the three subshapes of the final shape.</p>	<p>The student creates at least three different shapes made up of 2 trapezoids and a triangle with the total area of 50 cm^2.</p> <p>The student uses formulas to determine areas of shapes.</p> <p>The student can articulate why there must be additional solutions.</p> <p>The student realises there are many choices for the proportions of the three subshapes of the final shape.</p>

Wrapping a Prism

The surface area of a rectangular prism is close to 75 cm^2 .
What could the length, width and height be?

A large grid of dotted lines for drawing or writing, consisting of 15 columns and 20 rows of squares.

What's the point of this task?

There are different strategies students can use to figure out the surface area of a rectangular prism. Some might draw nets; others might visualise the faces in three dimensions. No matter how the problem is approached, students will need to recognise that rectangular prisms have 3 pairs of congruent opposite rectangular faces.

The total area selected was 75 square units, rather than 70 square units, for example, if students use whole number dimensions the surface area is an even number, not an odd number. The use of 75 forces students to think about the phrase 'close to'. Some students, no doubt, will use decimal or fractional dimensions, though.

Although some students will use formulas for the area of a rectangle, students who are uncomfortable with the formulas can use the grid background to help them.

Questions to facilitate the learning

- How many separate areas do you have to calculate to figure out the surface area?
- Could the dimensions have been whole numbers or did they have to be fractions or decimals?
- If you increase the length by 1 and width by 1, do you keep the same surface area or not?
- How do you know your results are correct?

Scaffolding the learning

- How many faces does the prism have? What do they look like?
- Why do some of the faces have to have the same dimensions? Do all of them?
- Suppose you made the height a big number. What does that mean about the length and width?
- Suppose you interchanged what you called the length and what you call the width. Would you have the same prism with the same surface area or a different one?

Extending the learning

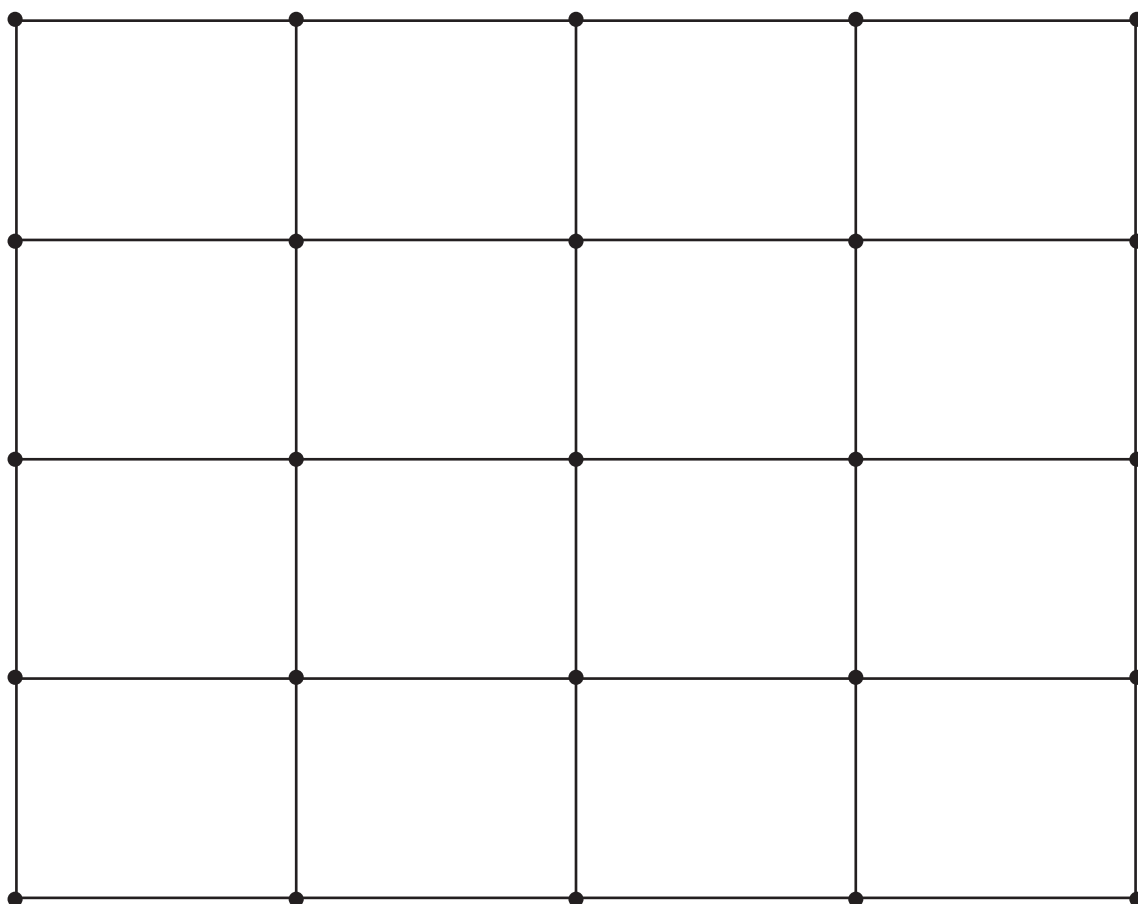
Students might use a triangular prism instead or the surface area could have been 30 cm^2 instead.

Rubric

Level 1	Level 2	Level 3	Level 4
<p>The student is unable to determine a set of rectangular prism dimensions with a reasonable surface area.</p> <p>The student cannot explain why the surface area could not have been exactly 75 cm^2 if whole number dimensions were used.</p> <p>The student is not able to describe how changes in a dimension affect changes in surface area.</p>	<p>The student determines a set of rectangular prism dimensions with a reasonable surface area.</p> <p>The student uses the grid to help him/her determine the surface area and does not use too much reasoning.</p> <p>The student cannot explain why the surface area could not have been exactly 75 cm^2 if whole number dimensions were used.</p> <p>The student is not able to describe how changes in a dimension affect changes in surface area.</p>	<p>The student determines a set of rectangular prism dimensions with a reasonable surface area.</p> <p>The student explains how the surface area is determined and uses some formulas in that determination.</p> <p>The student can explain why the surface area could not have been exactly 75 cm^2 if whole number dimensions were used.</p> <p>The student is somewhat vague on how changes in a dimension affect changes in surface area.</p>	<p>The student determines at least two or three sets of rectangular prism dimensions with a reasonable surface area.</p> <p>The student clearly explains how the surface area is determined and is efficient in that determination (e.g. using formulas and/or realizing that each area is repeated twice).</p> <p>The student can clearly explain why the surface area could not have been exactly 75 cm^2 if whole number dimensions were used.</p> <p>The student can clearly explain how changes in a dimension affect changes in surface area.</p>

Predicting Area

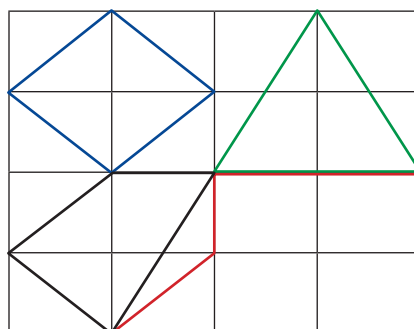
Make as many shapes as you can on the board where the vertices are positioned on the pegs and so there is exactly one peg inside the shape. How can you predict the area of the shape by knowing how many pegs are on its outside?



What's the point of this task?

Although most area formulas that students meet involve the use of lengths, widths, bases or heights of shapes, there is an area formula related to shapes on grids that never mentions these sorts of dimensions. It is called Pick's (or Pic's) formula and it says that the area of a shape on a geoboard is calculated by dividing the number of pegs on the perimeter by 2, adding the number of inside pegs and subtracting 1. Because the problem is set up requiring the number of inside pegs to be 1, the combined area of all of the shapes for this task will be half of the number of pegs on the perimeter.

There are several possible shapes, not just one.



Questions to facilitate the learning

- How were the shapes you created alike? How were they different?
- Would a 5-sided shape be possible? If not, why not? If so, what would it look like?
- What did you notice about the number of pegs on the boundary of your shapes?
- What did you notice about all the areas?

Scaffolding the learning

- How do you know that your shape cannot be too big?
- Could your shape be a rectangle with horizontal and vertical sides? Why or why not?

Extending the learning

Students might create shapes with exactly 2 pegs (or 3 pegs) inside and see how the areas do or do not change from when there is 1 peg inside.

Rubric

Level 1	Level 2	Level 3	Level 4
<p>The student is not able to create the required shapes.</p> <p>The student is not sure whether large shapes are possible.</p>	<p>The student creates a shape or two, but does not see what the shapes have in common.</p> <p>The student struggles to figure out the areas for some or all of the shapes.</p> <p>The student is not sure whether large shapes are possible.</p>	<p>The student creates a shape or two, but does not see what the shapes have in common.</p> <p>The student struggles to figure out the areas for some or all of the shapes.</p> <p>The student is not sure why very large shapes are not possible.</p>	<p>The student creates a sufficient number of shapes to see that the area of the shape is the same as half the number of pegs on the boundary.</p> <p>The student uses thoughtful strategies to figure out the areas of the shapes he/she creates.</p> <p>The student can explain why very large shapes are not possible.</p>

Equal for 10

$2x + 3$ is worth the same as another algebraic expression when $x = 10$ but not for other values of x .

What could the other expression be?

Are there other possibilities?

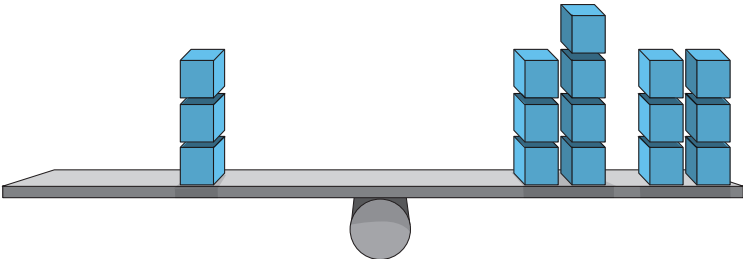
How could you use models to show that this is true?

What’s the point of this task?

It is essential for students’ algebraic development that they realise that different expressions can be worth the same amount when they are evaluated. In fact, solving an equation like $2x + 3 = 3x + 5$ is a way to determine when the two expressions $2x + 3$ and $3x + 5$ are worth the same.

Students might come up with expressions like $3x - 7$ or $x + 13$ or $\frac{x}{2} + 18$ which are worth the same as $2x + 3$ when $x = 10$, but not for other values of x . Other students might use an expression like $x + (x + 3)$ or $\frac{(4x + 6)}{2}$ which are other names for $2x + 3$ and are not only the same as $2x + 3$ when $x = 10$, but for all values of x .

The use of the models is to help students see that algebraic equalities can be modeled visually. For example, $2x + 3 = x + 13$ when $x = 10$ since the only way the sides below balance is if there are 10 cubes in each bag.



$2x + 3 = x + (x + 3)$ since the two lengths are the same.

x	x	3
x	x	3

Questions to facilitate the learning

- How did you figure out your other expression?
- Is it worth the same as $2x + 3$ for other values of x ? Why or why not?
- How does your model show that the expressions are equal when $x = 10$?

Scaffolding the learning

- What is the value of $2x + 3$ when $x = 10$?
- How else can you get 23 starting with a different number, not 10? How could you write what you described in words as an algebraic expression?

Extending the learning

Students might explore how the choices of expressions that are equal to $2x + 3$ when $x = 10$ are alike and different from those that are equal to $2x + 3$ when $x = 20$.

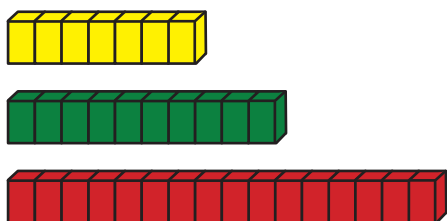
Rubric

Level 1	Level 2	Level 3	Level 4
<p>The student is unable to create an expression equal to $2x + 3$ when $x = 10$.</p> <p>The student struggles to explain how he/she created the expression.</p> <p>The student may use a model, but the model does not make it clear why the expressions have to be equal when $x = 10$.</p> <p>The student does not realise there are more possibilities.</p>	<p>The student creates at least one expression equal to $2x + 3$ when $x = 10$.</p> <p>The student struggles to explain how he/she created the expression.</p> <p>The student may use a model, but the model does not make it clear why the expressions have to be equal when $x = 10$.</p> <p>The student does not necessarily realise there are more possibilities.</p>	<p>The student creates several expressions equal to $2x + 3$ when $x = 10$.</p> <p>The student explains how he/she creates such expressions.</p> <p>The student uses a model to show why the expressions are equal when $x = 10$.</p> <p>The student clearly explains why there have to be even more possibilities, but does not contrast those that are always equal to $2x + 3$ with those that are only equal for $x = 10$.</p>	<p>The student creates several expressions equal to $2x + 3$ when $x = 10$.</p> <p>The student clearly explains how he/she efficiently creates such expressions.</p> <p>The student effectively uses a model to show why the expressions have to be equal when $x = 10$.</p> <p>The student clearly explains why there have to be even more possibilities and why some are or are not equal to $2x + 3$ for other values of x.</p>

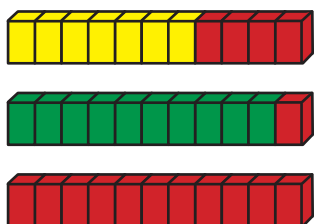
Building a Mean

One way to figure out the mean (average) of a set of data is to use cubes to represent the data and move the cubes around to make the data equal. The mean is the length of the equal pieces of data.

For example, the mean (average) of 7, 10 and 16 is 11 because:



It turns into what you see below when you rearrange the cubes.



Your job is to select 6 numbers so that the mean increases 4 of them but decreases 1 of them.

Use cubes to show you're right.

Try to find lots of possibilities.

What's the point of this task?

Students are introduced to how to use manipulatives to determine a mean in case this was new to them. But then they are asked to use what they see to solve a problem. One goal is for students to see that if 4 data values are increased and 1 is decreased, then the sixth data value must actually be the mean. Another goal is to realise that a fairly high value must be decreased if 4 values had to be increased. A third goal is to see that the total increase for the values that are increased matches the decrease for the value that is decreased. There are many possible solutions including, for example, 5, 5, 6, 7, 10, 27.

Questions to facilitate the learning

- Are there more low values or high values in your data? How do you know?
- What do you know about the value that was not increased or decreased?
- How did you figure out your six values?
- How did the total amount of increase for the 4 numbers compare to the decrease for the 1 number? Why did that happen?
- How do you know that there have to be HUNDREDS more answers?

Scaffolding the learning

- Could you use all equal values? Why or why not?
- Could you start with the cubes instead of the numbers? How?

Extending the learning

Students might explore what the data set could be if 3 values were less than the mean and 2 values were greater than the mean.

Rubric

Level 1	Level 2	Level 3	Level 4
<p>The student cannot determine a set of possible data values to meet the required criteria.</p> <p>The student cannot predict, in advance, why there must be one unusually large number and/or why one data value has to be the mean.</p>	<p>The student determines a set of possible data values to meet the required criteria.</p> <p>The student struggles to explain how he/she created the data set.</p> <p>The student cannot predict, in advance, why there must be one unusually large number and/or why one data value has to be the mean.</p> <p>The student struggles to create other solutions.</p>	<p>The student determines at least 2 sets of possible data values to meet the required criteria.</p> <p>The student explains how he/she created the data set.</p> <p>The student indicates that there must be one unusually large number and/or why one data value has to be the mean.</p> <p>The student can, if requested, determine more solutions, but does not have an efficient strategy to do so.</p>	<p>The student determines at least 3 or 4 sets of possible data values to meet the required criteria.</p> <p>The student clearly explains how he/she created the data set.</p> <p>The student recognises without referring to specific values why there must be one 'outlier' large number, why one data value has to be the mean and why the total increase matches the single decrease.</p> <p>The student explains a strategy for creating many more solutions, e.g. adding the same amount to all numbers or multiplying all numbers by the same amount.</p>