Take any prime number greater than 3, square it and subtract one.

Repeat several times.

Notice anything interesting? Convince yourself it always happens.

Assumptions and theorems Because of Result is. Process of proof

First of all, we can denote a prime number greater than 3 as p.

From "Take any prime number greater than 3, square it and subtract one"

We can get expression " q^2 -1 "

As we know a^2- b^2= (a+b)(a-b)

So q^2-1 = (q+1) (q-1)

FIRST

If we assume a group has three numbers, every group has one number, which must be divisible by 3. Eg. (1,2,3) (15,16,17) (209,210,211)

Take any integer "n"

There are three possibilities:

No remainder: In this case, n is divisible by 3

Remainder of 1: In this case, the next integer in the sequence (n + 1) must be divisible by 3

Remainder of 2: In this case, the integer two places ahead in the sequence (n + 2) must be divisible by 3

So, no matter what the remainder is when you divide one of the number by 3, one of the others must be divisible by 3.

Now we have three numbers (q-1, q ,q+1)

Q is a prime number, so q mustn't be divisible by 3, that means q-1 or q+1 must be divisible by 3.

SECOND

Since the prime q must be odd, q+1 or q-1 must be two consecutive even numbers.

We can replace q+1 and q-1 with n

q-1: 2n

q+1: 2n+2

(q+1)(q-1)

=2n(2n+2)

=4n^2+4n

=4n(n+1)

1. n is even number

   n can be 2k

make 2k into 4n(n+1) to replace n

4n(n+1)

= 4*2k(2k+1)

= 8k(2k+1)

Because k and (2k+1) are integer, so 8k(2k+1) must can be divisible by 8.

2. n is odd number

n can be 2k+1

make 2k+1 into 4n(n+1) to replace n

4n(n+1)

= 4(2k+1)(2k+1+1)

=4(2k+1)(2k+2)

=4(2k+1)2(k+1)

=8(2k+1)(k+1)

Because (2k+1) and (k+1) are integer, so 8(2k+1)(k+1) must can be divisible by 8.

In summary, for any even number n and multiplying it by consecutive even number must be divisible by 8. Which means (q+1)(q-1) must be divisible by 8

Summary

Back to the beginning "q"

We found $(q+1)(q-1)$ can divisible by 3 in first step

We found $(q+1)(q-1)$ can divisible by 8 in second step

$3*8=24$

So $(q+1)(q-1)$ can be divisible by $3*8$, which means can divisible by 24.

Because of $(q+1)(q-1)$ is same to q^2-1

So q^2-1 must can divisible by 24.